$$\therefore \Sigma\frac{1}{\sqrt{(R_aR_b)}}=\frac{1}{r}\,\Sigma\sqrt{\frac{(1-\sin\frac{1}{2}A)(1-\sin\frac{1}{2}B)}{(1+\sin\frac{1}{2}A)(1+\sin\frac{1}{2}B)}}$$

$$=\frac{1}{r}\,\frac{\Sigma(\cos\frac{1}{4}A-\sin\frac{1}{4}A)(\cos\frac{1}{4}B-\sin\frac{1}{4}B)(\cos\frac{1}{4}C+\sin\frac{1}{4}C)}{\pi(\cos\frac{1}{4}A+\sin\frac{1}{4}A)(\cos\frac{1}{4}B+\sin\frac{1}{4}B)(\cos\frac{1}{4}C+\sin\frac{1}{4}C)}$$

$$=\frac{1}{r}\,\frac{\Sigma\cos\frac{A+\pi}{4}\cos\frac{B+\pi}{4}\cos\frac{C-\pi}{4}}{\pi\cos\frac{A-\pi}{4}}$$

$$=\frac{1}{r}\,\frac{\Sigma(\cos\frac{1}{2}A+\cos\frac{1}{2}B-\cos\frac{1}{2}C)}{\Sigma\cos\frac{1}{2}A}=\frac{1}{r}.$$

Also solved by *H. C. WHITAKER*, and *G. B. M. ZERR*.

162. Proposed by J. D. PALMER, Providence, Ky.

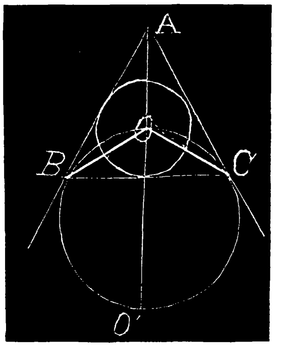
Given the distances from the vertices of a triangle, ABC, to the center of the circle, to construct the triangle.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

AO, BO, $CO=a$, b, c, respectively, where O is the center of the in-circle; BC, AC, $AB=x$, y, z, respectively. Let O_1 be the center of the ex-circle opposite A. Then

$$AO^2=\frac{(p-x)^2}{\cos^2\frac{1}{2}A},\ \text{where}\ p=\tfrac{1}{2}(x+y+z).$$

$$\therefore AO^2=\left[\frac{p-x}{p}\right]yz=yz-\frac{xyz}{p}=yz-4Rr=a^2.$$



Similarly, $BO^2=\left[\frac{p-y}{p}\right]xz=xz-4Rr=b^2.$

$$CO^2=\left[\frac{p-z}{p}\right]xy=xy-4Rr=c^2.$$

$$\therefore AO.BO.CO=\frac{\triangle\, xyz}{p^2}=4R^2r^2 \text{ or } 2\sqrt{(abc)}=4Rr$$

$$\therefore a^2+2\sqrt{(abc)}=yz,\ b^2+2\sqrt{(abc)}=xz,$$

$$c^2=2\sqrt{(abc)}=xy.$$

$$\therefore x[a^2+2\sqrt{(abc)}=y[b^2+2\sqrt{(abc)}]=z[c^2+2\sqrt{(abc)}]$$

$$=\surd\{[a^2+2\surd(abc)][b^2+2\surd(abc)][c^2+2\surd(abc)]\}.$$

This gives us the values of the sides.

Otherwise draw AO and produce AO to O_1 so that $OO_1=2\surd(bc/a)$. Upon OO_1 as diameter describe a circle. With O as a center and b as a radius describe an arc cutting the circle in B. Similarly, with O as center and c as radius, draw an arc cutting the circle in C. Join BC, AC, AB, then ABC is the triangle required. For O_1 is the ex-center opposite A by construction as follows:

$AO_1=p/\cos\frac{1}{2}A$. $\therefore$ $AOAO_1=yz=AO^2+2\surd(AO.BO.CO)$.

$\therefore$ $AO_1=AO+2\surd[(BO.CO)/AO]$.

CALCULUS.

121. Proposed by W. W. LANDIS. A. M., Professor of Mathematics and Astronomy, Dickinson College, Carlisle, Pa.

Solve the differential equation $\left[\dfrac{d}{dx}+b\right]^n y=\cos ax$.

Solution by LON C. WALKER, A. M., Petaluma High School, Petaluma, Cal., and LEWIS NEIKIRK, B. S., Boulder, Col.

$$\left[\frac{d}{dx}+b\right]^n y=\cos ax.$$

$\left[\dfrac{d}{dx}+b\right]^n$ has n roots each $=-b$.

$\therefore$ Comp. Factor $=e^{-bx}(c_1+c_2x+c_3x^2+c_4x^3+\ldots . c_nx^{n-1})$.

$$\frac{1}{\left[\dfrac{d}{dx}+b\right]^n}\cos ax=\frac{\left[\dfrac{d}{dx}-b\right]^n}{\left[\dfrac{d}{dx}-b^2\right]^n}\cos ax=\left\{\frac{b-\dfrac{d}{dx}}{a^2+b^2}\right\}^n\cos ax.$$

Let $n=1$. $\therefore$ $\dfrac{b-d/dx}{a^2+b^2}\cos ax=\dfrac{1}{a^2+b^2}(b\cos ax+a\sin ax)$

$$=\frac{1}{(a^2+b^2)^{\frac{1}{2}}}\left[\frac{b\cos ax+a\sin ax}{\surd(a^2+b^2)}\right]\ldots .(1).$$

Put $\theta=\cot^{-1}b/a$, then $\sin\theta=\dfrac{a}{\surd(a^2+b^2)}$, and $\cos\theta=\dfrac{b}{\surd(a^2+b^2)}$.

$\therefore$ (1) reduces to

$$\frac{b-d/dx}{a^2+b^2}\cos ax=(a^2+b^2)^{-\frac{1}{2}}(\cos\theta\cos ax+\sin\theta\sin ax)=(a^2+b^2)^{-\frac{1}{2}}\cos(ax-1\theta).$$